Small-Signal Analysis of Closed-Loop PWM Boost Converter in CCM with Complex Impedance Load

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Abstract—The following closed-loop transfer functions of the boost converter operating in continuous-conduction mode (CCM) supplying a complex impedance load are derived and analyzed: input-to-output voltage $M_{vcl}$ and reference-to-output $T_{pcl}$. The load of the boost dc-dc converter is composed of a series-connected resistance and inductance. The dynamic characteristics of the closed-loop boost converter with a third-order doublelead integral compensator are evaluated for different load inductances. The theoretically predicted results are validated through switching-circuit simulations using a suitable converter design example.

I. INTRODUCTION

A typical distributed power system involves a cascaded connection of a main dc voltage supply (or a dc feeder) linked to subsequent point-of-load (POL) dc-dc converters, each functioning at different operating conditions. The input impedance of the POL dc-dc converters exhibit characteristics such as (a) negative resistance up to the control bandwidth and (b) reactive impedance from low- to high-frequencies. Thus, the effective load impedance to the main dc voltage supply can be treated as a series combination of the load resistance $R_L$ and load reactance $X_L$ [1]-[5]. The present work considers a closed-loop boost converter with a series-connected resistance and inductance load.

The consequences of adding a high load inductance on the dynamics of the open-loop duty cycle-to-output transfer function of the boost converter were discussed in [5]. It was shown that the right-half plane zero $z_p$ moves closer to origin and enters the left-half plane as the value of the inductance was increased. The following conclusion was made:

$$\omega_{zp} = \omega_p = \begin{cases} 
\begin{align*}
\text{right-half plane} & : & L_L < L/(1 - D)^2 \\
\text{left-half plane} & : & L_L = L/(1 - D)^2
\end{align*}
\end{cases}$$

where $L_L$ is the load inductance, $L$ is the inductance at the input of the boost converter, and $D$ is the duty cycle.

This paper extends the previous work to analyze the characteristics of the main dc voltage supply controlled by a doublelead integral controller. The expressions for the closed-loop reference-to-output transfer function $T_{pcl}$ and the closed-loop input-to-output transfer function $M_{vcl}$ are derived and their frequency and time responses are carefully studied.

The objectives of this paper are as follows:

- To adopt the load inductance model to an already existing small-signal model of the closed-loop boost converter with resistive load.
- To develop the expressions for the closed-loop control-to-output $T_{pcl}$ and the closed-loop input-to-output voltage $M_{vcl}$ of the closed-loop boost converter in CCM.
- To analyze the effect of different values of the load inductance on the location of poles and zeros of the derived transfer functions.

II. OPEN-LOOP TRANSFER FUNCTIONS

The open-loop control-to-output transfer function $T_p$ and input-to-output voltage transfer function $M_v$ for the boost converter with complex impedance load has been derived in [5]. This section presents a discussion of the transfer functions in brief.

A. Open-Loop Control-to-Output Transfer Function $T_p$

Fig. 1 shows the small-signal model of the boost converter with an inductive load. The impedances in the model can be lumped and expressed as follows.

$$Z_1 = r + sL,$$ (2)

$$Z_2 = \left(\frac{r_C}{sC} + \frac{1}{sC}\right)((R_L + sL_L).$$ (3)

The open-loop control-to-output voltage transfer function of the boost converter with impedance load is obtained by setting
The following sections utilize the controller transfer function \( T_c \) in deriving the closed-loop transfer functions. As the zero shifts to the left half-plane altering the dynamics of the \( s \)-plane, while for higher values of the inductance \( L_L \), the zero shifts to the left half-plane altering the dynamics of the boost converter. As \( L_L \) increases, the ringing in its step response also increases.

**B. Open-Loop Input-to-Output Voltage Transfer Function \( M_v \)**

Using the analysis presented in [5], [6], [7], the expression for the open-loop input-to-output voltage transfer function in terms of the impedances is given by

\[
M_v(s) = \frac{v_o(s)}{i_c(s)} = \frac{Z_2(1-D)}{Z_1 + Z_2(1-D)^2},
\]

where the impedances \( Z_1 \) and \( Z_2 \) are as given in (2) and (3), respectively. Including the load inductance \( L_L \) into the transfer function results in an additional zero, whose location is always in the left-half of the \( s \)-plane.

III. CLOSED-LOOP TRANSFER FUNCTIONS

Fig. 2 shows the circuit of the closed-loop boost converter with third-order integral double-lead controller. The controller has a pole at the origin and two pole-zero pairs. The controller achieves a low dc steady-state error and allows a wide closed-loop bandwidth. Fig. 3 shows a small-signal model of closed-loop boost converter with the transfer functions of the feedback network, controller, and pulse-width modulator represented as \( \beta, T_c, \) and \( T_m \), respectively.
B. Closed-Loop Control-to-Output Transfer Function $T_{pcl}$

The closed-loop control-to-output transfer function is expressed as [6], [7]

$$T_{pcl}(s) = \frac{v_o(s)}{v_r(s)} \bigg|_{v_r, i_o=0} = \frac{T_c(s)T_m T_p(s)}{1 + \beta T_c(s)T_m T_p(s)}$$

(15)

where $T_c(s)$ is the voltage transfer function of the controller given in (11), $T_p(s)$ is the open-loop control-to-output transfer function given in the impedance form in (4), the $\beta$ is the feedback factor given by $\beta = \frac{R_B}{R_A + R_B}$, while the transfer function of the pulse-width modulator is $T_m = \frac{1}{V_{Tm}}$, where $V_{Tm}$ is the amplitude of the sawtooth waveform.

C. Closed-Loop Input-to-Output Transfer Function $M_{ccl}$

The small-signal model of the closed-loop boost converter required to determine the input-to-output transfer function $M_{ccl}$ is obtained by setting $v_r$ and $i_o$ to zero in Fig. 3. The expression for the closed-loop input-to-output transfer function in terms of the impedances is given by [6], [7]

$$M_{ccl}(s) = \frac{v_o(s)}{v_i(s)} \bigg|_{v_r, i_o=0} = \frac{M_o(s)}{1 + \beta T_c(s)T_m T_p(s)}$$

(16)

where $M_o$ is the open-loop audio susceptibility given in (5).

IV. RESULTS AND DISCUSSION

A. Design of Boost DC-DC Converter

A boost converter is designed for the following specifications: input dc voltage $V_I = 12$ V, switching frequency $f_s = 100$ kHz, minimum output power $P_{O_{max}} = 10$ W, and the dc output voltage is $V_O = 20$ V. Using the design equations presented in [6], the values of the boost inductor and capacitors for a nominal duty ratio of $D = 0.46$ are found to be: $L = 156 \mu$H, and $C = 6.8 \mu$F. The equivalent average resistance (EAR) considered in the inductor branch is $r = 0.24 \Omega$. The equivalent series resistance of the filter capacitor is $r_C = 0.111 \Omega$.

B. Compensator Design

An integral-double-lead controller is designed for the boost converter with the specifications discussed earlier. With a reference voltage $V_R = 2.5$ V, the voltage transfer function of the feedback network is calculated to be $\beta = \frac{V_d}{V_o} = \frac{R_B}{R_A + R_B}$. The resistances in the feedback network are assumed as $R_B = 620 \Omega$ and $R_A = 4.3 \Omega$. The $h$ parameters are calculated as: $h_{11} = \frac{R_B}{R_A + R_B} = 542 \Omega$, and $\frac{1}{h_{12}} >> R_L$ (can be neglected). The components of the integral-double-lead controller have values: $R_1 = 100 \Omega$, $R_2 = 100 \Omega$, $R_3 = 12.2 \Omega$, $C_1 = 4.7 \mu$F, $C_2 = 150 \mu$F, and $C_3 = 4.7 \mu$F.

The loop gain transfer function $T$ was evaluated and the following results were found for the designed boost converter with integral double-lead controller. The phase margin was $PM = 60.5^\circ$, gain margin $GM = 11.5$ dB, and a cross-over frequency $f_c = 4$ kHz was used. The theoretically obtained plots of the transfer functions were validated using Saber switching circuit simulations and the results are presented in the following section.

Fig. 4 shows the Bode magnitude and phase plot of the closed-loop reference-to-output transfer function $T_{pcl}$ for the different values of the load inductance. In [5], the authors verified the movement of the inherent RHP zero to the LHP as the load inductance was increased to $L_L \geq L/(1 - D)^2$. However, in the closed-loop reference-to-output transfer function, although the movement of the RHP zero towards the origin was observed, it crossed the origin only at extremely high values of the load inductance. Thus, a closed-loop boost converter with inductance load has characteristics similar to a closed-loop boost converter with resistive load for a wide range of load inductance values. In other words, the presence of complex load does not affect the dynamics of the boost converter. The theoretically obtained Bode plots were validated through Saber switching circuit simulations and the result is shown in Fig. 5. Fig. 6 shows the response of the output voltage for step changes in the duty cycle obtained using Saber circuit simulator. The rise time, overshoot, and the steady-state error for the response plots for all the inductions are identical.

D. Analysis of $M_{ccl}$

Fig. 7 shows the Bode magnitude and phase plots of the closed-loop input-to-output voltage transfer function for the different values of the load inductance. The magnitude and phase plots have identical characteristics for each of the three selected inductances, indicating the insensitivity of the closed-loop input-to-output transfer function to changes in the load inductance. Fig. 8 shows the simulation results obtained using Saber validating the theoretically predicted model of the boost converter. Further, Fig. 9 shows the output voltage response for small-signal changes in the input voltage obtained for the different load inductance values.

V. CONCLUSION

Small-signal analysis of the closed-loop boost dc-dc converter with an impedance load has been presented in this paper.
The feedback and control network consists of a third-order double-lead integral compensator. The small-signal transfer functions: the closed-loop control-to-output transfer function $T_{pcl}$ and the input-to-output voltage transfer function $M_{vcl}$ have been derived. Frequency-domain and time-domain characteristics of these expressions have been analyzed. It has been shown that the addition of the load inductance does not affect the closed-loop transfer functions as opposed to the open-loop transfer functions derived by the authors in [5]. In conclusion, as the load inductance of the closed-loop boost converter increases (a) the location of the inherent RHP zero in the control-to-output transfer function $T_{pcl}$ moves closer to the origin, and (b) the dynamics of the input-to-output transfer function remains unchanged.

**REFERENCES**


