End-to-End Throughput and Delay Analysis for IEEE 802.11 String Topology Multi-hop Network using Markov-Chain Model

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Abstract—This paper proposes the analytical expressions for the IEEE 802.11 string-topology multi-hop networks using Markov-chain model. For achieving that, the proposed analysis procedure includes two proposals, which are: (i) Bianchi’s Markov-chain model is modified for considering a relationship between the backoff timer and frame length, and (ii) the interferences, such as hidden node collision and carrier sensing, among network nodes are expressed by merging the proposed Markov-chain model and airtime expression. The analytical expressions are verified by comparisons with simulation results.

Index Terms—IEEE 802.11, string-topology, VANETs, multi-hop networks, end-to-end throughput, end-to-end delay, Markov-chain model, queuing theory

I. INTRODUCTION

Mathematical models are effective for comprehending the essence of network dynamics. This is because effects of system parameters to network dynamics and performances can be obtained explicitly from the mathematical model. Even though the quantitative accuracy is lost due to some idealizations and approximations, it is important to obtain the qualitative evaluations of the network performances as functions of system parameters [1]-[11].

Recently, the analyses of wireless multi-hop networks have been attention by many researchers [1]-[6]. The string-topology network is often selected as an analysis object because it is one of the fundamental and simple multi-hop network topologies. The string-topology networks are important and often considered in Vehicular Ad-hoc Networks (VANETs) [6]-[8]. Actually, many multi-hop network analysis techniques were developed from the string-topology multi-hop network analysis. However, the multi-hop network analyses, which are obtained until now, are not valid for long frame communications. The long frame communication, such as video streaming, over wireless multi-hop network is used in various situations, for example VANETs [8] and Wireless Multimedia Sensor Networks (WMSN) [9]. It is important to obtain fundamental analytical expressions for any frame lengths taking into account the interferences among network nodes.

This paper proposes the analytical expressions for the IEEE 802.11 string-topology multi-hop networks using Markov-chain model. The analytical expressions obtained in this paper give end-to-end throughput and delay at any frame length, and any offered load. For achieving that, the proposed analysis procedure includes two proposals, which are: (i) Bianchi’s Markov-chain model [11] is modified for considering a relationship between the backoff timer and frame length, and (ii) the interferences, such as hidden node collision and carrier sensing, among network nodes are expressed by merging the proposed Markov-chain model and airtime expression [1]-[4]. The analytical predictions agree with simulation results well, which show validity of the obtained analytical expressions.

II. MOTIVATION AND BACKGROUND

A. String-Topology Multi-hop networks

The string-topology networks are important and often considered in VANETs [6]-[8]. The VANETs require the data frames to be relayed via multiple hops between vehicles on the spot [7]. IEEE 802.11p specifications the physical- and MAC-layer features such that IEEE 802.11 could work in a vehicular environment. Because multi-hop vehicles are in line on the road, the vehicle-to-vehicle communications are often modeled by communications on string-topology multi-hop networks [6]. Though the string-topology network is a simple network topology, it is not easy to comprehend a network behavior. In this sense, it can be stated that analytical expressions of string-topology multi-hop network performances are useful and valuable.

B. Analysis for IEEE 802.11 String-Topology Multi-hop networks

There are many analytical procedure for wireless multi-hop networks [1]-[5]. One of the effective approaches for multi-hop network analysis is use of the ‘airtime’ expressions, which are time shares of the network-node state [1]-[4]. Because the channel-access situation can be expressed by using network-nodes airtimes, frame-collision probabilities induced by hidden node can be expressed with simple form. The individual node operations are associated by “flow constraint” conditions, which express the Network-layer property. For obtaining the maximum throughput of multi-hop networks, it is assumed in [1]-[3] that all the nodes have at least one frame in the transmission buffer. For expressing the operation with respect to each node in non-saturated condition, the concept of “frame-existence probability” was proposed by applying queuing theory to the airtime expressions [4].

It was, however, pointed out in [3] that these analyses are not valid for long-frame communications. This is because the relationship between the Backoff Timer(BT) -decrement time and frame-transmission one disappears by averaging the states.

C. Analysis for IEEE 802.11 WLAN based on Bianchi’s Markov-Chain Model

Bianchi proposed the Markov-chain model for expressing the BT-decrement in IEEE 802.11 Distributed Coordination Function (DCF) [11]. This analytical model has been considered as one of the seminal papers for the throughput model of IEEE 802.11 and has been extended in many subsequent studies [5], [10]. In the analytical procedure based on Bianchi model, however, it is assumed that collision
III. END-TO-END THROUGHPUT AND DELAY ANALYSIS FOR IEEE 802.11 STRING-TOPOLONY MULTI-HOP NETWORKS

Figure 1 shows the network topology considered in this paper. In this paper, H-hop string topology is considered. The analysis in this paper is based on the following assumptions [1]-[4]:

1. Each node is equipped with a single radio transceiver and all the network nodes use the same radio channel.
2. Only the source nodes (Node 0) generate fixed sized UDP data frames, payload size of which is P bytes, following Poisson distribution. The destinations of the frames generated by Node 0 is Node H.
3. Channel conditions of all the links are ideal. Namely, transmission failures occur only due to frame collisions.
4. Frame collisions between DATA and ACK frames and those among the ACK-frames transmissions can be ignored because ACK-frame length is shorter than DATA-frame length. [1]-[4]
5. Node i can transmit DATA and ACK frame only to Nodes i+1. Additionally, Nodes i+1 and i+2 can sense Node-i transmissions. Namely, Nodes i and i+3 are in the hidden node relationships [12]

A. Airtime

In this analysis, we use the airtime expressions, which are time shares of the node states with respect to each node [11]-[4]. The transmission airtime is the time share of frame transmissions, which includes both the successful- and the failure-transmission times. The transmission airtime of Node i is denoted as $X_i$.

The carrier-sensing airtime consists of frame-reception durations from the previous node and carrier-sensing durations from other nodes in the carrier-sensing range. Therefore, the carrier-sensing airtime is regarded as the sum of frame-transmission durations in all the nodes in the carrier-sensing range. The carrier-sensing airtime of Node i is:

$$Y_i = \sum_{j=i-2}^{i+2} X_j - \sum_{j=i-2}^{i+2} \left( \frac{X_jX_{j+3}}{1-X_{j+1}-X_{j+2}} \right) \frac{X_{j+2}X_{j+3}}{1-X_i}.$$  \hspace{1cm} (1)

When a node is in neither transmission state nor carrier-sensing states, the channel related with the node is idle. Namely, the channel-idle airtime is expressed as

$$Z_i = 1 - X_i - Y_i \hspace{1cm} (2)$$

B. Markov Chain Model

1) Transmission Probability: Figure 2 shows the Markov-chain model for BT-decrement and transmission states model of Node i. In Fig. 2, the value of contention window of s-th backoff stage is

$$W_s = \begin{cases} 2^s(CW_{\min} + 1) - 1 & 0 \leq s \leq L' - 1 \\ 2^{L'}(CW_{\min} + 1) - 1 = CW_{\max} & L' \leq s \leq L \end{cases}$$ \hspace{1cm} (3)

where $CW_{\min}$ and $CW_{\max}$ are minimum and maximum contention window, respectively, $L$ is the maximum backoff stage, which is the same as retransmission limit, and $L' = \log_2 \frac{CW_{\max} - 1}{CW_{\min} - 1}$. $h = DATA/\sigma$, where $DATA$ is the transmission time of the DATA and $F$ is the minimum backoff stage which satisfies $h \leq W_s$.

The Markov-chain model is defined in the DATA-frame transmission and BT-decrement state. In the previous analyses, the BT-decrement state and the DATA-frame-transmission one are considered separately. Therefore, the relationship between two states are ignored, which is a reason why the previous analyses are valid for long-frame communications [3]. By including the transmission state in Bianchi’s Markov-chain model as shown in Fig. 2, the relationship between the BT-decrement time and frame length can be expressed explicitly. In Fig. 2, the probabilities the transition probabilities among the states are

$$P[[s,t] = \begin{cases} 1, & s = 0, t = 1 \\ \gamma_i/(W_s + 1), & 0 < t \leq W_s \\ 1 - \gamma_i/(W_s + 1), & 0 < t \leq W_0 \end{cases}$$ \hspace{1cm} (4)

$$P[[0,0]] = 1/(W_0 + 1), \hspace{1cm} \sum_{s=0}^{L} \sum_{t=0}^{W_s} b[s,t] = b[0,0] = \sum_{s=0}^{L} \gamma_i h + \frac{W_s}{2} = 1.$$ \hspace{1cm} (5)
from which
\[ b[0, 0]i = 1/ \left( \sum_{s=0}^{L} \gamma_i^s \left( h + \frac{W_s}{2} \right) \right). \] (6)

Transmission probability of Node \( i \), which is in saturated condition, is obtained as
\[ T^r_i = \frac{\sum_{s=0}^{L} b[s, 0]i \gamma_i^s}{\sum_{s=0}^{L} \sum_{r=0}^{W_s} b[s, r]i} = \frac{\sum_{s=0}^{L} \gamma_i^s W_s}{2}. \] (7)

The operation in non-saturation condition is not considered in the proposed Markov-chain model. Transmission probability in both non-saturated and saturated condition is obtained as the product of transmission probability in saturated condition and frame-existence probability \([4], [10]\). In \([4]\), the frame-existence probability of Node \( i \) is expressed as
\[ q_i = \frac{\lambda_i (1 - V_i) \sigma U_i}{Z_i} = \frac{\lambda_i (1 - V_i) \sigma \sum_{s=0}^{L} \gamma_i^s W_s}{Z_i}. \] (8)

In (8), \( V_i \) is the buffer-blocking probability of Node \( i \). The expression of \( V_i \) is described in Section III-D. \( U_i \) is the average slot number of BT-decrement for one-frame transmission success of Node \( i \) and \( \lambda_i \) is frame-reception rate of Node \( i \). Because Node \( i \) relay the DATA frames, which are transmitted by Node \( i - 1 \), to Node \( i + 1 \) in the string-topology network, the frame-reception rate of Node \( i \) relates to throughput of Node \( i - 1 \). Therefore, the frame-reception rate of Node \( i \) is expressed as
\[ \lambda_i = \frac{E_{i-1}}{P}. \] (9)

In (9), \( E_i \) is throughput of Node \( i \), which is expressed as
\[ E_{i-1} = X_{i-1}(1 - \gamma_{i-1})^P \frac{P}{T}. \] (10)

\[ T = DIFS + DATA + SIFS + ACK, \] where \( DIFS \) is the duration of the DIFS, \( SIFS \) is the duration of the SIFS, \( ACK \) is the transmission time of the ACK. The reception rate of Node \( 0 \) is network offered load \( O \). Namely, \( E_{i+1} = O \). The transmission probability of Node \( i \) is obtained as
\[ \tau_i = q_i \gamma_i^s = \frac{\lambda_i (1 - V_i) \sigma \sum_{s=0}^{L} \gamma_i^s W_s}{Z_i}. \] (11)

Because \( Z_i \) is a function of transmission airtime, transmission probability is a function of transmission airtime and collision probability and frame-reception rate.

2) Collision Probability: In string-topology networks, two types of frame collisions with carrier-sensing range nodes and hidden nodes occur. Because these two collisions are disjoint events, the frame-collision probability of Node \( i \) is expressed as
\[ \gamma_i = \gamma_C + \gamma_H, \] (12)

where \( \gamma_H \) is hidden node collision probability of Node \( i \) and \( \gamma_C \) is carrier-sensing nodes collision probability of Node \( i \).

The carrier-sensing range node collisions occur only when
\[ \gamma_C = 1 - \prod_{j=i-1}^{i+2} \left( 1 - \tau_j \right). \] (13)

A hidden node collision occurs when Node \( i \) starts to transmit a frame during the Node \( i + 3 \) transmitting a DATA-frame. When Node \( i + 3 \) is in data-transmission state at the instant that the BT of Node \( i \) is zero, hidden node collision occurs. Therefore, the collision probability of this type of hidden-node collision is expressed as
\[ \gamma_{(1)}_{Hi} = \frac{\frac{h \sigma}{T} X_{i+3} + q_{i+3} Z_{i+3}}{1 - X_{i+1} - X_{i+2}} \sum_{s=0}^{L} b[s, t_i+3]. \] (14)

Additionally, a hidden-node collision also occurs when Node \( i + 3 \) starts to transmit a frame during the Node-\( i \) transmission. Namely, when “BT of Node \( i + 3 \) is smaller than \( h \)” and “Nodes \( i + 4 \) and \( i + 5 \), which are the carrier-sensing range nodes of Node \( i + 3 \), are not in the transmission state” at the instant that the BT of Node \( i \) is zero, the collision occurs. Therefore, this type of hidden-node collision probability is expressed as
\[ \gamma_{(2)}_{Hi} = \frac{\frac{h \sigma}{T} X_{i+3} + q_{i+3} Z_{i+3}}{1 - X_{i+1} - X_{i+2}} \sum_{s=0}^{L} b[s, t_i+3] \sum_{r=0}^{L} b[s, r_i+3] - \frac{1}{1 - X_{i+1} - X_{i+2}}. \] (15)

Because the two types of hidden node collisions are disjoint events, the hidden node collision-probability is
\[ \gamma_{Hi} = \gamma_{(1)}_{Hi} + \gamma_{(2)}_{Hi}. \] (16)

From (8), (12), (13), (14) and (15), collision probability is a function of transmission airtime and collision probability, transmission probability and frame-reception rate.

C. Flow Constraint in Multi-hop Networks

The transmission airtimes of network nodes are fixed by taking into account Network-layer properties. Because each airtime depends on the states of neighbor nodes, transmission airtimes of network nodes are associated with Network-layer properties.

When the retransmission counter reaches the retransmission limit \( L \), the frame is dropped following the DCF policy. Additionally, the frame is dropped when the buffer of receiver is full. Therefore, the throughput of each node should satisfy
\[ E_i = E_{i-1}(1 - \gamma_i^{L+1})(1 - V_i). \] (17)

The relationship in (17), which is called as the flow-constraint condition, expresses the network-layer property. By eliminating \( E_i \) and \( P \) from (9), (10) and (17), we have
\[ X_i = \frac{\lambda_i (1 - V_i) T (1 - \gamma_i^{L+1})}{1 - \gamma_i} = \lambda_i (1 - V_i) TR_i. \] (18)

where \( R_i \) is the average number of transmission attempts for one-frame transmission success of Node \( i \).

D. Buffer-Blocking Probability

The buffer queue is modeled by using airtime expression and queueing theory. Figure 3 shows the buffer queuing model of
Node $i$, where $K$ is the buffer size and $\mu_i$ is the frame-service rate of Node $i$. The frame-service time is defined as the average time interval between the instant when a frame reaches the top of the transmission-node buffer and the one when the frame is transmitted successfully to the next node. Namely, the frame-service time is the same as MAC access delay. From [4], the frame-existence probability in whole time with respect to Node $i$ is expressed as

$$Q_i = \frac{X_i + q_i Z_i}{1 - Y_i} = \frac{X_i + q_i Z_i}{X_i + Z_i}. \quad (19)$$

Because the ratio of the sum of the BT-freezing and BT-decrement durations to transmission duration is $\frac{Q_i(1 + q_i Z_i)}{X_i}$, the frame-service time of Node $i$ is expressed as

$$D_{M_i} = TR \left(1 + \frac{X_i + q_i Z_i}{X_i}\right) = TR \frac{(X_i + q_i Z_i)}{X_i (X_i + Z_i)} = \frac{(TR_i + \sigma U_i)(1 - V_i)}{X_i + Z_i}. \quad (20)$$

Therefore, the frame-service rate of Node $i$ is expressed as $\mu_i = \frac{1}{D_{M_i}}$. The utilization rate of Node $i$ is obtained as

$$\rho_i = \frac{\lambda_i}{\mu_i} = \frac{X_i + q_i Z_i}{(X_i + Z_i)(1 - V_i)} = \frac{Q_i}{1 - V_i}. \quad (21)$$

From the buffer-queueing model in Fig. 3, the steady state probability that the Node $i$ has $k$ frame is expressed as

$$\pi_{i,k} = \frac{\lambda_i}{\mu_i} \pi_{i,k-1} = \left(\frac{\lambda_i}{\mu_i}\right) \pi_{i,0} = \rho_i^k \pi_{i,0}. \quad (22)$$

The sum of all the buffer-state probability should be one, namely

$$\sum_{k=0}^{K} \pi_{i,k} = \frac{(1 - \rho_i^{K+1})\pi_{i,0}}{1 - \rho_i} = 1. \quad (23)$$

From (22) and (23), therefore, we have

$$\rho_i^k = \frac{\rho_i^k - \rho_i^{k+1}}{1 - \rho_i^{K+1}}. \quad (24)$$

Because the buffer-blocking probability is the same as the steady state probability that the Node $i$ has $K$ frame, namely

$$V_i = \pi_{i,K} = \frac{\left(\frac{Q_i}{1 - V_i}\right)^K - \left(\frac{Q_i}{1 - V_i}\right)^{K+1}}{1 - \left(\frac{Q_i}{1 - V_i}\right)^{K+1}}. \quad (25)$$

From above expression, buffer-blocking probability is a function of transmission airtime and collision probability and frame-reception rate.

From (9), (11), (18), (12) and (25), $5H$ algebraic equations are obtained. These equations contain $5H$ unknown parameters, which are $X_i$, $\tau_i$, $\gamma_i$, $A_i$, and $V_i$, for $i = 0, 1, 2, \ldots, H - 1$. It is possible to fix the $5H$ unknown parameters and the offered loads are given. In this paper, Newton’s method is applied for obtaining the $5H$ unknown parameters.

E. End-to-End Delay

In the string-topology multi-hop networks as shown in Fig. 1, the end-to-end delay is defined as the duration from the instant when a frame is generated at the source node to the one when the frame is received at the destination node, which is the sum of the single-hop transmission delay from Node 0 to Node $H - 1$. Each single-hop transmission delay consists of two parts, which are the MAC access delay and the queuing delay.

By using the buffer-state probability, queueing delay of Node $i$ is expressed as

$$D_{Q_i} = \sum_{k=1}^{K} \left[\frac{D_{M_i}}{2} + (k - 1)D_{M_i}\right] \pi_{i,k} = \frac{D_{M_i} \rho_i (1 + \rho_i - 2K + 1) \rho_i^K + (2K - 1) \rho_i^{K+1}}{2(1 - \rho_i)(1 - \rho_i^{K+1})}. \quad (26)$$

Transmission delay of Node $i$ is obtained as

$$D_i = D_{M_i} + D_{Q_i}. \quad (27)$$

Because the end-to-end delay is the sum of the single-hop transmission delay from Node 0 to Node $H - 1$, the end-to-end delay of string-topology network is

$$D = \sum_{i=0}^{H-1} D_i. \quad (28)$$

IV. SIMULATION VERIFICATION

In this section, the validity of the obtained analytical expressions are discussed by comparing with simulation results. Table I gives system parameters based on the IEEE 802.11a standards. An original simulator, which was implemented by authors, was used in this paper because it is necessary to obtain the detailed data from simulations. The credibility of the simulator is confirmed by quantitative agreements of simulator is compared with the results from ns-3 simulator [13].

Figure 4 shows end-to-end throughputs of six-hop network with one-way flow as a function of offered load and $P = 200$ bytes. It is seen from Fig. 4 that show quantitative agreements with simulation ones. At $O_1 = 1.15$ Mbps, the maximum throughput is obtained. The end-to-end throughput is saturated at $O_1 = 1.6$ Mbps. It is confirmed analytical
expression presented in this paper can express the network behavior both in non-saturated condition and saturated one.

Figure 5 shows maximum throughputs of eight-hop network as a function of payload size. It is seen from Fig. 5 that all the results from proposed analysis, analysis in [1], and simulations show quantitative agreements in short-frame length. The throughput obtained from [1] has difference from the simulation result as the payload size increases. The proposed analysis gives accurate maximum throughput prediction regardless of the payload size. This result shows that the importance to consider the relationship between the BT decrement time and frame-transmission one for achieving the accurate predictions.

Figure 6 shows end-to-end delay of Flow 1 in six hop network as a function of offered load under the condition of $P=1300$ bytes. In Fig 6, analytical results from the proposed analytical expressions and from the model in [5] are plotted. It is seen from Fig 6 that analytical results from [5] have differences from simulation results. This is because the interferences, such as hidden node collision and carrier sensing, with respect to each node can be considered in the proposed analytical model.

V. CONCLUSION

This paper have proposed the analytical expressions for the IEEE 802.11 string-topology multi-hop networks using Markov-chain model. For achieving that, Bianchi's Markov-chain model is modified for considering a relationship between the backoff timer and frame length. Additionally, the interferences, such as hidden node collision and carrier sensing, among network nodes are expressed by merging the proposed Markov-chain model and airtime expression. The analytical expressions are verified by comparisons with simulation results.

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