In this paper, we propose a variety of the time-domain shooting method for finding steady state responses of nonlinear circuits in order to implement it on SPICE-like simulators. Furthermore, the proposed method is extended to the design of Class E amplifier. In the numerical examples, the proposed method is demonstrated using the latest version of Berkley SPICE.

1. Introduction

Steady state analysis of nonlinear circuits had been studied in the communities of microwave, circuits and systems, and communications. Recently, this study is again spotlighted due to increasing consumption rise of communication devices, computer, and electronics with wireless functions. Numerical methods for finding the steady state responses of nonlinear circuits are roughly categorized into the frequency-domain, the time domain shooting, and the mixed domain methods [1]. The frequency-domain method represents the steady state responses by Fourier series and then the circuit equation is solved at each frequency component. The time-domain shooting method is essentially a method for solving two point boundary value problem, because the solution is constrained so that the initial values of state variables are the same with the responses at the time equals to the period of a input [3]. The mixed domain method utilizes both advantages of the time- and frequency-domain methods.

In this paper, we proposes a variety of the time-domain shooting method and implement it on SPICE-like simulators. Furthermore, the proposed method is extended to parameter determination of Class E amplifier [5]. Class E amplifier has a possibility for high-speed and high-efficiency operation of power amplifier and some applications to power electronics and communications are expected [4]. Therefore, the SPICE netlist based implementation of the proposed procedure would be valuable for the developments of these systems.

2. Incomplete Shooting Method

2.1. Time-Domain Shooting Method

Consider a general network containing an arbitrary number of linear/nonlinear components. The MNA formulation of such a network can be written as [2],

\[ C \dot{x} + G x + P(x) = b(t) \]  \hspace{1cm} (1)

where \( x \) is the vector consisting of node voltage, independent voltage source current, linear inductor currents, nonlinear capacitor charge, nonlinear inductor flux, and currents and voltages of nonlinear components; \( C \) and \( G \) are matrices describing the lumped memory and memoryless elements of the network, respectively; \( b(t) \) is a vector consisting of the independent voltage/current sources; \( P(x) \) is a nonlinear function matrix which describes the nonlinear elements of the circuits.

Assuming the independent voltage/current sources with a period \( T \), we find the steady-state responses of the circuit written by (1). The steady-state solution satisfies

\[ F(x(0)) = x(0) - x(T) = 0. \]  \hspace{1cm} (2)

Hence, finding the steady state response is reduced to the nonlinear optimization problem for determining the initial solution of \( x(0) \) with respect to \( x(T) \).

The Newton Raphson method is applied to solving this problem[3]:

\[ F(x^{j+1}(0)) = x^{j}(0) - x^{j}(T) \]
\[ + \left( I - \frac{\partial x(T)}{\partial x(0)} \right)_{x=x^{j}(0)} (x^{j+1}(0) - x^{j}(0)). \]  \hspace{1cm} (3)

Each element of the Jacobian \( J = I - \partial x(T)/\partial x(0) \) is calculated by giving a small perturbation to the network and carrying out the transient analysis. Suppose that a small perturbation \( \eta \) is appended to the initial values \( x_j(0) \) as \( x_j(0) + \eta \).
Then, the initial value problem of (1) is solved. Each element of the Jacobian is calculated as

\[ J(i, j) = \begin{cases} 
1 - \frac{\Delta x_i \eta(T)}{\eta} & (i = j) \\
-\frac{\Delta x_i \eta(T)}{\eta} & (i \neq j) 
\end{cases} \]  

(4)

where \( \Delta x_i \eta(T) \) is variation in \( x_i(T) \) after a small perturbation \( \eta \) is appended to \( x_j(0) \).

### 2.2. Incomplete Method

The time-domain shooting method finds steady state responses repeating transient analysis of a network. Thus, one may think that this method is easily implemented on SPICE-like simulators. However, simulators do not provide all informations of the MNA matrix. Node voltage, inductor current, capacitor voltage, some kinds of voltages and currents for nonlinear elements are only available for initial conditions. We develop the frame work for finding the steady state responses on this restriction.

Assume \( z \in \mathbb{R}^{N \times N} \) to be a vector of node voltages and inductor currents in \( x \) of (1). Then, the steady state condition is given by

\[ F(z(0)) = z(0) - z(T) = 0. \]  

(5)

The shooting method provided in the previous subsection is carried out on only (5) to find the steady state solution. In a circuit, memory elements such as inductor and capacitor are included in the inside and outside of semiconductor devices. The memory elements exist in the outside of devices are more dominant on the steady-state condition than ones in the inside. Namely, when the voltage and current waveforms of the memory elements exist in the outside of devices satisfies (5), the inside of the devices almost reaches the steady state. Therefore, we can obtain the almost steady-state responses by the time-domain shooting method, nevertheless the condition (5) is not complete.

### 2.3. Implementation

The incomplete shooting method sometimes fails to converge into the appropriate solutions. To improve the convergence, we use a damping parameter \( \beta \) \((0 < \beta \leq 1)\) as

\[ z^{j+1}(0) = z^j(0) - \beta \left( I - \frac{\partial z(T)}{\partial z(0)} \right)^{-1} \times \left( z^j(0) - z^j(T) \right). \]  

(6)

If \( \beta = 1 \), then, the iteration (6) is corresponding to the Newton method.

The implementation of the incomplete shooting method on Berkeley SPICE is summaries as:

1. Carry out operating point and transient analysis. Get the transient responses at \( t = T \) from SPICE output file.
2. Rewrite '.ic' line and 'ic=' values for each inductor description using the response at \( t = T \).
3. Add a small perturbation to one of initial voltages in '.ic' line or an initial inductor current.
5. Repeat steps 3 and 4 until all elements of the Jacobian are calculated.
6. Carry out the damping Newton method (6).
7. Repeat from steps 3 to 6 until a stopping condition is satisfied.

### 3. Application to Design of Class E Amplifier

The incomplete shooting method provided in the previous section is Incorporated into the design of the Class E amplifier [5]. A topology of basic class E amplifier is shown in Fig. 1, where the circuit consists of input voltage \( V_D \), dc-feed inductor \( L_C \), MOS switch S, shut capacitor \( C_S \) to the switch, a series resonant circuit composed from inductor \( L_0 \) and capacitor \( C_0 \), and output resistor R. In order to attain the high-efficiency, minimize all losses that occur when switching the transistor, which demands that the drain-source voltage when the switch closes is zero. Furthermore, it is necessary that the derivative of the switch voltage is also equal to zero at the
switching moment [4]. As a result, the constraints of Class E amplifier are obtained by

\[ v_s(T) = 0 \] (7)
\[ \frac{dv_s}{dt} \bigg|_{t=T} = 0. \] (8)

Figure 2 shows the typical waveform of the switch voltage \( v_s \) of the Class E amplifier of Fig. 1 To obtain such a waveform, one must adjust the passive components so that (7) and (8) are satisfied. The constraints must be valid on the steady state. Thus, the incomplete steady state condition (5) is enforced together with (7) and (8).

We also apply the damping Newton method to solving the composite problem. First, the derivative of the switching voltage is approximated by the backward Euler method:

\[ \frac{dv_s}{dt} \bigg|_{t=T} = \frac{v_s(T) - v_s(t_p)}{h}. \] (9)

where \( h = T - t_p \) is a time step size of the numerical integration.

When appending a small variation \( \eta \) into \( x_j(0) \), elements of the Jacobian associated with (7) and (8) are respectively calculated as

\[ J(N + 1, j) = \frac{\Delta v_{s,x_j}(T)}{\eta}, \] (10)
\[ J(N + 2, j) = \frac{\Delta v_{s,x_j}(T) - \Delta v_{s,x_j}(t_p)}{h}, \] (11)

where \( \Delta v_{s,x_j}(T) \) and \( \Delta v_{s,x_j}(t_p) \) are variation in \( v_s(T) \) and \( x_s(t_p) \), respectively, when appending a small perturbation \( \eta \) to \( x_j(0) \).

On the other hand, assuming \( M \) design parameters, elements of the Jacobian when appending a small variation \( \eta \) into a design parameter \( \lambda_k \) (\( k \leq M \)) are written by

\[ J(N + 1, j + k) = \frac{\Delta v_{s,\lambda_k}(T)}{\eta}, \] (12)
\[ J(N + 2, j + k) = \frac{\Delta v_{s,\lambda_k}(T) - \Delta v_{s,\lambda_k}(t_p)}{h}, \] (13)

where \( \Delta v_{s,\lambda_k}(T) \) and \( \Delta v_{s,\lambda_k}(t_p) \) are variation in \( v_s(T) \) and \( x_s(t_p) \), respectively, when appending a small perturbation \( \eta \) to \( \lambda_k \).

When the number of the design parameters is larger than 2, the Jacobian \( J \) becomes a rectangular matrix. Then, we use QR decomposition [7] rather LU one in order for update of the solution.

4. Numerical Results

4.1. CR amplifier

The incomplete shooting method was applied to the CR amplifier shown in Fig. 3, where LEVEL 1 of the device model was used. Using NGSPICE, which is the latest version of Berkley SPICE [8], we calculated the steady state response. Figure 4(a) shows the waveforms obtained from the proposed method. For a comparison, the transient analysis
was carried out until 0.1 [sec.]. We show the responses from 0.998 [sec.] to 0.1 [sec.] as shown in Fig. 4(b). The waveforms obtained from the proposed method are in agreement with the transient responses which reach to the steady state.

4.2. Class E amplifier

To design the Class E amplifier shown in Fig. 1, we define the following parameters [5]:

1) $\omega = 2\pi f$
2) $\omega_0 = 2\pi f_0 = 1/\sqrt{L_0C_0}$.
3) $Q = \omega L_0/R$.
4) $A = f_0/f = \omega_0/\omega$.
5) $B = C_0/C_S$.
6) $H = L_0/L_C$.

Specification: $f = 1.0$ MHz, $V_D = 5.0$ V, $R = 5.0$ $\Omega$, $Q = 10.0$, and $H = 0.001$, was given, and $A$ and $B$ were selected as the design parameters. The procedure provided in Sect. 3 was applied to this problem, where the switch $S$ was treated as the ideal switch with 0.16 $\Omega$ on-resistor. As a result, $A = 0.941728$ and $B = 0.6304492$ were obtained and all passive elements are determined such as $L_C = 7.957747\mu$H, $C_S = 5.693287\mu$F, $L_0 = 7.957747\mu$H, $C_0 = 3.589328\mu$F, and $R = 5.0\Omega$. Figure 5 shows the steady state waveforms of the Class E amplifier with these values of passive elements. We can see that the circuit shown in Fig. 1 certainly behaves as a Class E amplifier.

5. Conclusions

The incomplete time-domain shooting method for finding steady state responses of nonlinear circuits has been proposed, which is intended to implement it on SPICE-like simulators. In the numerical examples, the steady state responses can be obtained from the NGSPICE implementation. Furthermore, this method is extended to the design of Class E amplifier and the passive components including in this circuit are automatically determined. Prospect of Class E amplifier will gain the proposed design procedure significantly.

References