NEW PARTITION-BASED FILTERS FOR SUPPRESSING MIXED HIGH PROBABILITY IMPULSE AND GAUSSIAN NOISES IN IMAGES

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ABSTRACT

In this paper, we propose a new partition-based filter for removing mixed high probability impulse and Gaussian noises by extraction of the signals. The proposed filter sorts the elements of sample vector and extracts the signals around the median from sorted signals. The extraction vector is classified into one of \( M \) partitions, and a particular filtering operation is then activated. The output of the filter is estimated by the current pixel and the extraction vector. Carrying out the simulation, we illustrate the peak signal to noise ratio of the proposed filter, and show that it is effective in removing mixed high probability impulse and Gaussian noises.

1. INTRODUCTION

Nonlinear filters have been applied in treating impulse noise in signal restoration applications [1]-[4]. Among the voluminous class of nonlinear filters, those based on order statistics have demonstrated excellent robustness properties, and they are conceptually simple and easy to implement.

Recently, partition-based filtering has been studied for removal noise[1]. The filter for each partition is optimized only for data falling into that partition. As a partition-based filtering, partition-based median type (PBM) filter[1] was proposed for suppressing mixed noises in images. With the PBM filter, observed sample vector is classified into one of \( M \) partitions, and a particular filtering operation is then activated. The partitioning is based on the differences between the current pixel and the outputs of center weight median (CWM) filters[3]. The estimation at each location is formed as a linear combination of the outputs of those CWM filters and the current pixel. The weights of combination are optimized using the constrained least mean square (LMS) algorithm. Since impulse noise is removed by CWM filter and Gaussian noise is removed by the linear combination, PBM filter performs satisfactorily in removing mixed noises. However, in the case of high probability impulse noise, the performance of CWM filter is significantly reduced. Since the outputs of CWM filter are used by the partition and the estimation, the performance of the partition and the estimation are deteriorated as reduction high probability impulse noise is deteriorated.

In this paper, we propose a new partition-based filter for removing mixed high probability impulse and Gaussian noises by the extraction of the signals. The proposed filter sorts the elements of sample vector and extracts the signals around the median from sorted signals. Since the signals near maximum and minimum include almost impulse noise, the impulse noise can be removed from the sample vector. In addition, the current pixel can be judged whether it is corrupted with impulse noise or not. If the pixel is judged to be corrupted, the pixel is replaced with a median of extraction vector before the estimation. By using the current pixel and the extraction vector, the outputs of the proposed filter are estimated. In this process, the removal of Gaussian noise is achieved. Since impulse noises are removed by the extraction regardless of the probability of impulse noise, the estimation of each location is improved for mixed high probability impulse and Gaussian noises. Moreover, the neural network(NN)[2] is used for removal of Gaussian noise. Since the extraction vector have a nonlinear property, NN may have better characteristics than the linear combination for noise reduction in the system. Carrying out the simulation, we illustrate the peak signal to noise ratio (PSNR) of the proposed filter, and show that it is effective in removing mixed high probability impulse and Gaussian noises.

2. PBM FILTER [1]

A block diagram of PBM filters is shown in Fig.1. The PBM filter consists of CWM filters, partition index detection and the weights with the LMS algorithm. The PBM filter has a detection based median filters to remedy the above situation with the crisp switching operations. Specifically, a difference vector is formed on the differences defined between the current pixel and the outputs of CWM filters with variable center weights. The difference vector is divided into \( M \) mutually exclusive cells. Each of cells has a filtering operation formulated as a linear combination of the current pixel and the outputs of CWM filters. The optimal weights are derived by training the filter over a reference image with the constrained LMS algorithm.

PBM filter can suppress mixed noises by the partitioning and the linear combination with CWM filter. However, in the case of high probability impulse noise, the noise reduction of the PBM filter is declined. That is because the ability of removal of the CWM filter is reduced for high probability impulse noise. Therefore, it is a very important problem to consider partition-based filtering for mixed high probability impulse and Gaussian noises.
3. PROPOSED METHOD

In this paper, a partition-based filter for suppressing mixed high probability impulse and Gaussian noises is proposed. The input signal \( x(c) \) is given by

\[
x(c) = \begin{cases} 
  x_o(c) + g : \text{prob. } 1 - p \\
  0 & : \text{prob. } p/2 \\
  255 & : \text{prob. } p/2
\end{cases}
\]  

(1)

where \( x_o(c) \) is the original image, \( p \) is impulse noise ratio, and \( g \) is Gaussian noise with the zero-mean and the standard deviation \( \sigma \). At each location \( c \), a filter window of size \( 2n + 1 \), and the sample vector \( x(c) \in R^{2n+1} \) observed via the filter window is given by

\[
x(c) = \{ x(c-n), \cdots, x(c), \cdots, x(c+n) \}
\]  

(2)

where \( x(c) \) is the current pixel.

A topology of the proposed filter is shown in Fig.2. The filter consists of sort-extraction, partition index detection and NN.

### 3.1. EXTRACTION OF THE SIGNALS

In the PBM filter [1], the performance of noise removal for high probability impulse noise declines. In the proposed filter, the elements of sample vector are sorted and around the median of them are extracted for the reduction of impulse noise. Since the extraction vector includes few impulse noises, the following process has few influences of impulse noise on the restoration of images.

The extraction process is illustrated in Fig.3. First, \( x(c) \) is sorted and \( x_o(c) \in R^{2n+1} \) is obtained. Next, \( E_l \) pixels are cut off from the minimum of pixel, and \( E_h \) pixels are cut off from the maximum of pixel as shown in Fig.3. Therefore, \( E \) pixels are cut off from \( x_o(c) \), and \( z(c) = \{ z_0(c), \cdots, z_{S-1}(c) \} \in R^S \) is obtained, where \( E \) is the number of the cut signals, that is \( E = (E_l + E_h) \), and \( S \) is the number of extraction signals, namely, \( S = (2n+1) - E \). Since the signals near maximum and minimum of \( x_o(c) \) include almost impulse noise, the impulse noise can be removed by the extraction of the signals. If \( E \) is small, impulse noise may remain. On the other hand, in the case where \( E \) is large, the number of signals for the estimation of the output of the filter decreases.

### 3.2. REPLACEMENT OF CURRENT PIXEL

In the case of preservation of edge, the information of the current pixel is important for image restoration. Therefore, the information of the current pixel is necessary for the estimation. However, if the current pixel is corrupted by impulse noise, we have to carry out suitable processing.

The prediction of the current pixel \( x(c) \) is carried out by using \( z(c) \). If \( z_0(c) \leq x(c) \leq z_{S-1}(c) \), it is regarded that the current pixel \( x(c) \) is never impulse noise. On the other hand, in the case of \( x(c) < z_0(c) \) or \( x(c) > z_{S-1}(c) \), the current pixel is predicted to be corrupted by the impulse noise and replaced with the median of \( z(c) \). The replacement is defined by

\[
z_e(c) = \begin{cases} 
  x(c) & : \text{if } z_0(c) \leq x(c) \leq z_{S-1}(c) \\
  \text{median}\{z(c)\} & : \text{otherwise}
\end{cases}
\]  

(3)

where \( z_e(c) \) is used to estimate the filter output.

### 3.3. PARTITION INDEX DETECTION

The filter for each partition is optimized only for data falling into that partition. In this paper, difference vector \( d(c) \) is obtained from \( z(c) \) and \( z_e(c) \). The difference vector \( d(c) \) is given by

\[
d(c) = \{ d_0(c), d_1(c), \cdots, d_l(c) \} \in R^l
\]  

(4)

In (4), the elements of \( d(c) \) is calculated as follows:

\[
d_0(c) = |z_0(c) - z_e(c)| \\
d_l(c) = |z_{S-1}(c) - z_e(c)|
\]
where \( m = (S - 1)/2 \) is the median index of \( z(c) \). By using difference vector \( d(c) \), the extraction vector is classified into one of \( M \) partitions. The difference vector \( d(c) \) reveals the information about the likelihood of edge for the sample vector. For example, let us consider the difference \( d_0(c) \) and \( d_4(c) \). If \( d_0(c) \) or \( d_4(c) \) is large, then we can predict the sample vector is spread and included in the edge. On the other hand, \( d_0(c) \) and \( d_4(c) \) are small, the sample vector is in the smooth area.

Interval endpoints \( q_{k,v} \) \((k = 0, 1, \ldots, 4, v = 0, 1, \ldots, L)\) are used for the determination of the index of used filter \( p(c) \) as shown in Table 1. The partition index \( p(c) \) is defined by

\[
\begin{align*}
p(c) &= \{p_0(c) + p_1(c) \cdot L + \cdots + p_4(c) \cdot L^4 + 1\} \\
p_0(c) &= v : if \quad q_{k,v} \leq d_0(c) < q_{k,v+1} \\
&\quad (k = 0, \ldots, 4, v = 0, \ldots, L - 1) \quad (7)
\end{align*}
\]

where \( L \) is the number of interval for differences, \( p(c) = 1, 2, \ldots, M \) and the total number of partition \( M \) is \( M = L^5 \).

### 3.4. NEURAL NETWORK

In PBM filter, the estimation of each location is carried out by the linear combination. However, the linear combination cannot adapt flexibly to change of the feature of the sample vector. In this paper, the estimation at each location is carried out by the NN[2] instead of the linear combination. Since the extraction vector has a nonlinear property, NN may have better characteristics than the linear combination for the extraction vector. Therefore, NN is superior to linear combination in Gaussian noise reduction.

### 4. SIMULATION RESULTS

The performance of the proposed filters has been evaluated by the simulations. The performance of restoration is quantitatively measured by the peak signal to noise ratio(PSNR). Girl (256 × 256, 8bits) is used as a reference image. In the training, the image corrupted by mixed impulse \((p = 20\%)\) and Gaussian \((\sigma = 20)\) noises on the reference image is made as the input image of the filter. And the output image of the filter is compared with the reference image. The coefficients of the filter are updated by back propagation algorithm using the compared result.

In filters, \( 5 \times 5 \) windows are used, then \( n = 12 \) and window size is \( 2n + 1 = 25 \). The interval endpoint values used for partition index detection listed in Table 1, where \( L = 4 \) and the total number of NN is \( M = 4^5 = 1024 \). \( E \) is set to 6 and 10.

\[\text{Lenna}(256 \times 256 , 8bits) \text{ is used as processing images.}\]

This image is corrupted by mixed impulse noise \((p = 10 \sim 70[\%])\) and Gaussian noise \((\sigma = 20)\).

The performance of the proposed filter of the image restoration in mixed impulse and Gaussian noises removal is compared with those of other filters, including median filter, progressive switching median(PSM) filter[4] and PBM filter[1]. Here, the reference image of PBM filter is Girl corrupted by mixed impulse \((p = 20\%)\) and Gaussian \((\sigma = 20)\) noises.

Fig.4 shows the results of filtering Lenna corrupted by the mixed Gaussian \((\sigma = 20)\) and impulse noises with different noise ratios. Here, the proposed filter is named **sorting neural network** (S-NN) filter. In Fig.4, the PSNR of the S-NN filter exceeds other filters over \( p = 20\% \). For the low probability impulse noise, the S-NN filter is inferior to PBM filters in PSNR. Since the extraction of the S-NN filter reduces some pixels, the data for the restoration of the image is decreased. On the other hand, for the high probability impulse noise, the S-NN filter is superior to other filters. Since other filters cannot remove impulse noise for high probability impulse noise, the PSNR of other filters is declined. However, the S-NN filter can remove impulse noise with the extraction, and the PSNR of the S-NN filter is kept for high probability impulse noise. For low probability impulse noise, the PSNR of the S-NN filter for \( E = 6 \) is similar to that for \( E = 10 \). On the other hand, for high impulse noise, the PSNR of the S-NN filter for \( E = 10 \) is better than that for \( E = 6 \). This is because the extraction for \( E = 10 \) can remove impulse noise for high probability
Fig. 5 shows the results of filtering Lenna corrupted by mixed impulse noise. The extraction for $E = 6$ can remove impulse noise for about 6/25% probability of impulse noise. Therefore, the extraction for $E = 6$ remains some impulse noises for high probability impulse noise. Since the extraction for $E = 10$ removes almost impulse noise regardless of the probability of impulse noise, the PSNR of the S-NN filter for $E = 10$ is almost constant. As $E$ is increased, the data of the restoration of images is decreased. However, the disadvantage is covered by the nonlinear property of the NN.

In this paper, we have proposed a new partition-based filter for removing mixed high probability impulse and Gaussian noises by extraction vector. The extraction has reduced impulse noise and the performance of the estimation was improved. Furthermore, the partitioning by using the current pixel and the extraction vector has been introduced. In the simulation, the proposed filter has demonstrated superior performance in suppressing mixed high probability impulse noise and Gaussian noises. Especially, it is noteworthy that the proposed filter has shown the noticeable difference from other methods in PSNR as well as the image quality.

**5. CONCLUSIONS**

In this paper, we have proposed a new partition-based filter for removing mixed high probability impulse and Gaussian noises by extraction vector. The extraction has reduced impulse noise and the performance of the estimation was improved. Furthermore, the partitioning by using the current pixel and the extraction vector has been introduced. In the simulation, the proposed filter has demonstrated superior performance in suppressing mixed high probability impulse noise and Gaussian noises. Especially, it is noteworthy that the proposed filter has shown the noticeable difference from other methods in PSNR as well as the image quality.

**6. REFERENCES**


