Impulse Noise Detector Using Mathematical Morphology

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Abstract—Switching schemes have been studied for removing impulse noise. As a switching scheme, pixel-wise median of the absolute deviations from the median (PWMAD) detector was proposed. Since PWMAD detector uses only a single parameter, it is easy to optimize a parameter. However, PWMAD detector can not detect noisy pixels accurately when noisy pixels exist in neighborhood of edge pixels. In this paper, we propose an impulse noise detector using mathematical morphology. We use mathematical morphology in order to improve noise detection in neighborhood of edge pixels. Moreover, the proposed method requires only a single parameter by using mathematical morphology. Carrying out the simulation, we will illustrate the noise detection ratio of the proposed method, and show that it is more accurately to detect noisy pixels than PWMAD detector without increasing parameters.

I. INTRODUCTION

In image processing, a median filter has been widely used for removing impulse noise, since a median filter is quite effective for noise removal and edge preservation[1]-[9]. However, a median filter tends to modify both noisy pixels and undisturbed good pixels[4], [5].

Recently, switching schemes have been studied for removing impulse noise in images[2]-[8]. These schemes detect whether the current pixel is degraded by impulse noise at each pixel. Then, filtering is activated for pixels that are detected as noisy pixels, while good pixels are kept. The main problem in the detection scheme is the optimization of tradeoff between noise removal and edge preservation. In order to solve this problem, filtering methods exist: four thresholds in [3], a set of fuzzy rules and membership functions in [6]. Additionally, multistate version in [3], fuzzy filters[7], and the neural network method [8], are based on previous training. However, it is difficult to optimize many parameters.

As a detection scheme without a number of optimizing parameters and previous training, pixel-wise median of the absolute deviations from the median (PWMAD) detector was proposed[5]. PWMAD detector uses a single parameter and only a simple median filter. PWMAD detector calculates the absolute deviation which is the absolute difference between input image and the median of it. The absolute deviations are large for noisy pixels and edge pixels. In PWMAD detector, a median filter is used to extract only edge pixels from the image for the absolute deviations. By eliminating edge pixels in the image for the absolute deviations, only noisy pixels can be detected. Therefore, it is important to extract positions of edge

\[ X_{ij} = \{ x_{i-K,j-K}, \ldots, x_{ij}, \ldots, x_{i+K,j+K} \} \] (2)

In this paper, the image degraded by impulse noise is defined as follow,

\[ x_{ij} = \begin{cases} s_{ij} : \text{prob. } 1 - p \\ 255 : \text{prob. } p/2 \\ 0 : \text{prob. } p/2 \end{cases} \] (1)

where \( i, j \) is the pixel coordinates of an image, \( s_{ij} \) is the original image. The image \( x_{ij} \) is degraded by impulse noise whose value is 255 with a generation probability \( p/2 \) and 0 with a generation probability \( p/2 \). At each location \( i \) and \( j \), the size of the filter window \( W \) is \( (2K+1) \times (2K+1) \). The sample \( X_{ij} \) observed via the filter window is defined as follow:
The procedure for PWMAD detector consists of calculations of absolute deviations and median filtering. At first, PWMAD detector calculates the image for the absolute deviations $d_{ij}$. The image for the absolute deviations $d_{ij}$ is defined as follow,

$$d_{ij} = |x_{ij} - \text{median}(X_{ij})|$$  \hspace{1cm} (3)

where median($X_{ij}$) is the median of $X_{ij}$. The image for the absolute deviations is shown in Fig. 1. Figure 1(a) shows the image for the absolute deviations from the original image. Figure 1(b) shows the image for the absolute deviations from the degraded image by impulse noise. Here, Figure 1(a) and (b) are reversed. In Fig. 1(a), the absolute deviations are large at edge pixels. Moreover, in Fig. 1(b), the absolute deviations are large at edge pixels and noisy pixels. In the image for the absolute deviations, edge pixels and noisy pixels correspond to sequential points and isolate points, respectively. In PWMAD detector, a median filter is used to separate edge pixels from noisy pixels without parameters. By eliminating edge pixels in the image for the absolute deviations, only noisy pixels can be detected. The procedure of PWMAD detector is applied iteratively. The procedure in an iterative manner is shown as follow,

$$D_{ij}^{(n)} = \{d_{ij-(k-1)}^{(n)}, \ldots, d_{ij}^{(n)}, \ldots, d_{ij+(k+1)}^{(n)}\}$$  \hspace{1cm} (4)

$$d_{ij}^{(0)} = |x_{ij} - \text{median}(X_{ij})|$$  \hspace{1cm} (5)

$$d_{ij}^{(n+1)} = |d_{ij}^{(n)} - \text{median}(D_{ij}^{(n)})|$$  \hspace{1cm} (6)

where $n$ is the number of iterations. After $N$ iterations, $d_{ij}^{(N)}$ is obtained. Noisy pixels are detected by comparing $d_{ij}^{(N)}$ with a threshold $T$.

$$f_{ij} = \begin{cases} 1, & d_{ij}^{(N)} \geq T \\ 0, & d_{ij}^{(N)} < T \end{cases}$$  \hspace{1cm} (7)

If $f_{ij}$ is 1, then $x_{ij}$ is regarded as a noisy pixel. On the other hand, if $f_{ij}$ is 0, then $x_{ij}$ is regarded as an original pixel. Since PWMAD detector uses only a single parameter, it is easy to optimize a parameter.

In PWMAD detector, a median filter is used to extract edge pixels. The median filter has the edge shift problem[9]. When a noisy pixel exists in the neighborhood of the edge, the edge shift is generated. Figure 2 shows an example of the edge shift. Figure 2(a) shows vertical edge. Figure 2(b) shows vertical edge degraded by impulse noise. Figure 2(c) shows the median filter suppresses noise. However, the edge shift is generated in Fig. 2(c). The edge shift is generated when two or more impulse noise with the highest value is added to the edge in the lower level, or when two or more impulse noise with the lowest value is added to the edge in the higher level. Consequently, because of the edge shift, positions of extracted edge pixels do not correspond to correct positions of edge pixels. In PWMAD detector, the accuracy of noise detection is low for the edge shift.

III. PROPOSED METHOD

Figure 3 shows the construction of the proposed method. The procedure of the proposed method consists of calculations of the absolute deviations and mathematical morphology. The opening in mathematical morphology is used to extract edge pixels. Because of using the opening, edge pixels can be extracted without the edge shift. Moreover, the opening does not require optimizing parameters. Consequently, noisy pixels are detected accurately without increasing parameters.

A. Mathematical Morphology[10]

Mathematical morphology is based on erosion and dilation. Erosion and dilation are expressed as follow,

$$\text{Erosion} : \quad A \ominus B = \cap_{b \in B} (A - b)$$  \hspace{1cm} (8)

$$\text{Dilation} : \quad A \oplus B = \cup_{b \in B} (A + b)$$  \hspace{1cm} (9)

where $A$ is an input image, $B$ is the filter window, $\ominus$ and $\oplus$ are expressed as erosion and dilation operators, respectively. Based on these two operators, the opening and the closing are...
In the proposed method, erosion and dilation have a dual relationship[10]. That is, a reduced region by erosion is equal to an expanded region by dilation. Therefore, edge pixels are restored to their original positions for the opening. The edge shift is not generated. By using the opening, positions of extracted edge pixels correspond to those in the image for the absolute deviations. Therefore, noisy pixels are detected with high accuracy.

The performance of noise detection is quantitatively measured by the false detection probability($\frac{\text{the number of false detection pixels}}{\text{all pixels}} \times 100\%$) and the missed detection probability($\frac{\text{the number of missed detection pixels}}{\text{the number of noisy pixels}} \times 100\%$). The missed detection means that a noisy pixel is regarded as an original signal pixel. The false detection means that an original signal pixel is regarded as a noisy pixel.

The performance of restoration is evaluated by Peak Signal-to-Noise Ratio (PSNR). PSNR is defined by

$$\text{PSNR} = 10 \log_{10} \frac{255^2}{\frac{1}{H \times L} \sum_{i=1}^{H} \sum_{j=1}^{L} (Z(i,j) - S(i,j))^2} \text{[dB]}$$

where $H \times L$ is the size of an image, $Z$ is a restored image, $S$ is an original image. The parameters are determined empirically, setting iteration count to $N = 1$, the size of the filter window to $W = 3 \times 3$, and threshold $T = 45$.

A. Performance of Noise Detection

The performance of noise detection by the proposed method is compared with those by conventional methods, including PSM filter (the part of noise detection)[2] and PWMAD detector[5]. Parameters of conventional methods are tuned respectively. Figure 5 shows the false detection probability and the missed detection probability for each detector. In Fig. 5, the missed detection probability of the proposed method is equal to or higher than that of PSM. When the proposed method extracts only edge pixels but also some noisy pixels, noisy pixels are eliminated in the image for the absolute deviations.

### IV. Simulation Results

In the simulation, the effectiveness of the proposed method is demonstrated by real processing results. We use LENNA, BOAT, WOMAN (256 × 256, 8 bits image) as test images. In the part of image restoration, we use PSM filter algorithm[2]. PSM filter algorithm restores the noisy pixels using only original pixels in the filter.

**Example**

- **Opening**: $A \circ B = (A \otimes B) \oplus B$ (10)
- **Closing**: $A \bullet B = (A \oplus B) \ominus B$ (11)

Erosion outputs the minimum value in the filter window. Dilation outputs the maximum value in the filter window. Therefore, the opening does not require optimizing parameter. In addition, the opening removes convex impulse noise in an image, and the closing removes concave impulse noise in an image[10]. Figure 4 shows an example of mathematical morphology. Figure 4(a) shows an original image. Figure 4(b) shows an image applied erosion to Fig. 4(a). Figure 4(c) shows an image applied the opening to Fig. 4(a). In Fig. 4(b), edge is shifted and an isolate point is removed. In Fig. 4(c), positions of edge pixels correspond to those in the image for the absolute deviations, only noisy pixels are large. The opening is used to extract the edge pixels for the absolute deviations, the value of noisy pixels and edge pixels are removed.

**B. Impulse Noise Detection by Mathematical Morphology**

In the proposed method, we detect noisy pixels using mathematical morphology. As shown in section 2, in the image for the absolute deviations, the value of noisy pixels and edge pixels are large. The opening is used to extract the edge pixels from the image for the absolute deviations. By eliminating edge pixels in the image for the absolute deviations, only noisy pixels can be detected. The difference image $d'_{i,j}$ is defined as

$$d'_{i,j} = |d_{i,j} - OPEN(d_{i,j})|$$

where $OPEN(d_{i,j})$ is the opening of $d_{i,j}$, Eq (12) is called the top-hat transformation[10]. Noisy pixels are detected by comparing $d_{i,j}$ with a threshold $T$.

$$f_{ij} = \begin{cases} 1, & d'_{i,j} \geq T \\ 0, & d'_{i,j} < T \end{cases}$$

If $f_{ij}$ is 1, then $x_{ij}$ is regarded as a noisy pixel. On the other hand, if $f_{ij}$ is 0, then $x_{ij}$ is regarded as an original pixel. Since proposed method uses only a single parameter, it is easy to optimize a parameter.

In the proposed method, erosion and dilation have a dual relationship[10]. That is, reduced region by erosion is equal to expanded region by dilation. Therefore, edge pixels are restored to their original positions for the opening. The edge shift is not generated. By using the opening, positions of extracted edge pixels correspond to those in the image for the absolute deviations. Therefore, noisy pixels are detected with high accuracy.
deviations. However, the false detection probability of the proposed method is over 2% than that of PSM in each impulse noise ratio. This is because, the proposed method can separate edge pixels from noisy pixels. In Fig. 5, both the false detection probability and the missed detection probability of the proposed method is equal to or lower than those of PWMAD detector. Because of edge extraction by the opening in the proposed method, the edge shift is not generated. Therefore, the accuracy of edge extraction by the proposed method is better than that of edge extraction by PWMAD detector. From Fig. 5, the proposed detector shows better results than PWMAD method without increasing parameters.

B. Performance of Restoration

The performance of image restoration by the proposed method is shown. The part of image restoration in conventional method is PSM filter algorithm[2].

Figure 6 shows restoration results of each method. In Fig. 6(b) and (d), the proposed method preserves edge and details more than PSM filter. This is because, the proposed method separates noisy pixels from edge pixels and decreases false detection at edge. In Fig. 6(c) and (d), the proposed method removes impulse noise especially at edge more than PWMAD detector. This is because the accuracy of edge extraction by the proposed method is improved compared with that of PWMAD detector.

Fig. 7 shows the result of PSNR. In Fig. 7, PSNR of the proposed method is higher than those of other methods in each case. Since the proposed method reduces the false detection keeping the performance of the un-detection, PSNR is improved.

V. Conclusion

In this paper, we have proposed an impulse noise detector using mathematical morphology. Since the proposed method requires only a single parameter, it is easy to optimize a parameter. We use the opening in order to extract edge pixels from the image for the absolute deviations. By using the opening, positions of extracted edge pixels correspond to positions of the edge pixels in the image for the absolute deviations. The proposed method can extract edge pixels more accurately than PWMAD detector without increasing parameters. Hence, the performance of noise detection is improved. Carrying out the simulation, we will illustrate the noise detection probability of the proposed method, and show that it is effective to detect impulse noise.

REFERENCES